kant and precolonial mathematics

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Abstract

In his posthumously published essay "Mathematical Ideas in Early Philippine Society," Ricardo Manapat presented the case of the Angono Petroglyphs, a set of Neolithic (about 3,000 BC) cave drawings located in the hills of Angono, Rizal, a province in the south of Manila. Jesus Peralta, a famed anthropologist discovered that there are 127 figures in the petroglyphs clearly discernible as integral units. These figures demonstrate various depictions of symmetry such as triangle and parallelogram, scaling, and proportions demonstrating an intuitive mathematical work and prehistoric forms of abstractions. Manapat posited in his article that old Tagalogs assigned 'limit numbers' such as a thousand *yuta* (100,000,000) against the idea of mathematical infinity. He also reports that whatever goes beyond this number is no longer conceivable.

In this paper, I aim to read the precolonial Philippine mathematical system through the lens of Kant's concepts of pure intuition and mathematics. Within this dialectic, I claim that intuitive precolonial ethnomathematics works parallel with Kant's thoughts on intuition as the revelatory ground for mathematical concepts which thereby serve as means for determining anything unfounded and arbitrary. The precolonial math system, constituted by ideas of symmetry, integral units, shapes and limit numbers, are mere forms of appearances sensible to Neolithic people in their spatio-temporal magnitudes. By claiming ethnomathematical observation as accessible to the precolonial's pure intuition alone, I look to locate a possible site where these two dynamic systems, i.e., Kant's epistemic rigor and precolonial ethnomathematics, would successfully converge.

Keywords:
Angono petroglyphs, mathematics, intuition, appearances, pure reason

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To absorb a process of change
in the gesture of a fearless arm
emotion that moves and grows:
forms being formed to forget
the finiteness of beginnings.

-- Maningning Miclat, *Why a Mural?*

Suppositions

Intuition remains a contentious subject in the history of philosophy. Intuition is seen either a source of pure rational cogency or a senseless derivative of knowledge. However, the German thinker Immanuel Kant projected that intuition can be viewed not only as pure cognition but also, juxtaposing sensibility, an aid to produce mathematical knowledge. Kant charted interesting points that influenced future philosophical systems. Thus, the *first supposition*: this paper tours us to the cartography of Kant's critical philosophy specifically the concept of intuition (Transcendental Aesthetic) and its role in providing mathematical truths from his oeuvres *The Critique of Pure Reason* and the *Prolegomena*.

This paper will also examine a variant of mathematical knowledge. First conceived in 1977 by Brazilian educator Ubiratan D' Ambrosio, *ethnomathematics* is a kind of mathematical knowledge practiced by cultural, ethnic or indigenous groups. Given its non-Western and non-Eurocentric description, this paper will also extend the definition to cover ethnomathematical practices in precolonial Philippines. Thus, the *second supposition*: If Kant presupposes that his variant of intuition is a key player in formulating mathematical truths, then, can we also posit the same case for precolonial mathematics? Thus, this paper will enlighten us about three things: a) the dynamics of intuition in Kantian philosophy; b) ethnomathematical knowledge; and c) the role of intuition (Kant) in precolonial ethnomathematical knowledge.

Kant's Doctrine of Intuition

The publication of the *Critique of Pure Reason* in 1781 marked a turning point in Kant's philosophical career. While most German universities that time embraced the systematic Wolffian philosophy, Kant repudiated his former philosophical conscience, that is to say, his *preliminary* and yet *original* awakening from Wolff's dogmatism two years earlier than his famous provocation "interruption from dogmatic slumber" in his shorter opus *Prolegomena to Any Future Metaphysics*.[1] This awakening places Kant's *Critique* as a middle term to the budding philosophy of Wolff not to mention the arising *rationalism-empiricism* debate within the continental thought. The *Critique*, as it turns out, combined two inconsistent theories flagging the way to a more refined philosophical system.

This combination is traceable in Kant's treatment of intuition in his first *Critique* (Transcendental Aesthetic). For Kant, intuition is "that which it [cognition] relates immediately to them [objects]."[2] This relation takes place whenever the object is given and has the capacity to affect our mind. Kant called this *givenness* of the object as *sensibility*.[3] Sensation occurs when the object affects our cognition thus, as an empirical intuition.[4] Kant notes that empirical intuition ensues only in the appearance of an object corresponding to a particular sensation (he called it *matter*) but there are manifolds of appearances set to be ordered in a certain relation (he called it *form*) organized in an immediate intellection. The matter of all appearances, that which we can perceive sensibly, is handed to us through *a posteriori* perception while the form of all appearances, to which our mind orders or categorizes, is handed to us in *a priori*. Thus, cognition is possible through the workings of sensibility and intuition. The former gives the object and the latter provides the relation. With these processes, cognition produces the *concept* emanating from our understanding as *thought*.[5] Kant argues that

> [without] sensibility no object would be given to us, and without understanding none would be thought. Thoughts without content are empty, intuitions without concepts are blind. It is thus just necessary to make the mind's concepts sensible (i.e., to add an object to them in intuition) as it is to make its intuitions understandable (i.e. to bring them under concepts). Further, these two faculties or capacities cannot exchange their functions. The understanding is not capable of intuiting anything, and the senses are not capable of thinking anything.[6]

This popular Kantian precept outlines the strong relation of thought to intuition. Without the other, the entire cognition collapses but Kant charges that they are sundry and 'cannot exchange their functions.' By posing a complete difference, Kant pounced on rationalists and empiricists who are guilty of this predicament. For example, a rationalist would take thought to be the same as intuition as primary sources of intellection; while an

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[1] Paul Guyer and Allen Wood (trans. and ed.), "Introduction," in *Immanuel Kant's Critique of Pure Reason* (Cambridge: Cambridge University Press, 1997), 23.

[2] Immanuel Kant, *Critique of Pure Reason*, trans. and eds. Paul Guyer and Allen Wood (Cambridge: Cambridge University Press, 1997), 172; [Henceforth, *Critique*].

[3] Ibid.

[4] Ibid.

[5] Ibid.

[6] Ibid., 193.

empiricist would take sensibility, as the material representation of thought, as the only true source of knowledge. This, then and again, brings us to the distinction between *a priori* and *a posteriori* knowledge. The harmonious relation between sensibility and intuition, that is to say, sensibility's provision of object and intuition's ordering of relation, can only be regarded in *a priori* knowledge. In this light, Kant proposes a science of *a priori* sensibility viz. the *transcendental aesthetic:*

> In the transcendental aesthetic we will therefore first isolate sensibility by separating off everything that the understanding thinks through its concepts, so that nothing but empirical intuition remains. Second, we will then detach from the latter everything that belongs to sensation, so that nothing remains except pure intuition and the mere form of appearances, which is the only thing that sensibility can make available *a priori*.[7]

The transcendental aesthetic leads us to pure intuitions, namely, that of space and time. For Kant, space is not an empirical concept derived from external experiences but is a necessary representation; it is not discursive but appears essentially singular in the manifold of all spaces and also uniquely *a priori*.[8] Space, by its *a priori* cognition, only shows mere representations of sensibility "which can be different in different people."[9] But to access the *thing in itself* in space through intuition is impossible. It can only provide *synthetic a priori judgments*, for an instance, geometric truths by which we can grasp alone via mental constructions of lines and from which we can hash out the said judgment. Kant also argues that time is also a derivative of pure intuition. He insists that time has one dimension, apodictic in its principle and like space, not discursive.[10] In addition, Kant reasons that the original representation of time is given unlimited but "every magnitude of an object" can only be determined through limitation. In this light, time, without the feature of the infinite, can only be known and determined by pure intuition – together with an object of cognition – in a given magnitude. Things in space and time are perceivable insofar they are empirical representations.[11] That which is given, Kant assumes that all possible intuitions need to pass empirical cognition in order to become objects of experience. Here, the object, as subjected to empirical cognition, transforms as a *phenomenon*, which for Kant is not merely appearance but appearing. The French philosopher Gilles Deleuze has a say on this,

[7] Ibid., 174.
[8] Ibid., 175.
[9] Ibid., 178.
[10] Ibid., 179.
[11] Ibid., 254.

> We can see that, in Kant, phenomenon means not appearance, but appearing. The phenomenon appears in space and time: space and time are for us the forms of all possible appearing, the pure forms of our intuition or our sensibility. As such, they are in turn presentations; this time, *a priori* presentations. What presents itself is thus not only empirical diversity in space and time, but the pure *a priori* diversity of space and time themselves. Pure intuition (space and time) is the only thing which sensibility *presents a priori*.[12]

Deleuze argues, as far as sensibility and intuition present *a priori* truths, that intuition is not representation. He notes that the prefix 're-' (in representation) implies an active taking up of what is already *given* – a *synthesis* of what already is presented. Therefore, knowledge is not anymore a synthesis of representations. In this joint, Kant recognizes "*Imagination* [as] the faculty for representing an object even without its presence in intuition."[13] Imagination saves intuition *in potentia*, by representing it in thought. Kant made a series of excellent examples. For an instance, we cannot think of a line without drawing it in thought, we cannot think of a circle without describing it, and we cannot represent time without drawing a straight horizontal line in progression (or two hands of an analogue clock).

Kant's doctrine of intuition combines the sensible and the intelligible in the vein that one does not overwhelm the other. In this framework, Kant resolved the conflict that concerned philosophers before him. This doctrine clears our judgments from rationalism and empiricism and thus exposes the deeper sense of cognition in establishing intuitive relation among objects under proper categories of thought. Kant exposed the mutuality between sensibility and intuition in all amenities of cognitive power. The next section of this paper will uncover another 'cognitive power' of intuition namely mathematical cognition.

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[12] Gilles Deleuze, *Kant's Critical Philosophy*, trans. Hugh Tomlinson and Barbara Habberjam (Minneapolis: University of Minnesota Press, 1999), 8.

[13] Kant, *Critique*, 256. Deleuze in this regard considered imagination, understanding and reason as active faculties as sources of real representations (Deleuze, *Kant's Critical Philosophy*, 8-9).

Intuition and Mathematical Cognition

Kant was able to sketch a grander view of mathematical cognition. For example, Gordon Brittan, a Kantian scholar, claims that Kant precisely knows the history of mathematics by way of comparing it to philosophy but his writings about the former is scattered in his notes that anyone who wants to wrestle with the topic should reconstruct his thoughts.[14] He also adds that if one surveys the *Critique*, the discussion on mathematics surfaces in the *Introduction* and toward the end of the section *Transcendental Method*.

Briefly, there are two assertions that Kant made to support his philosophy of mathematics: first, mathematical cognition is *synthetic a priori*; second, mathematical cognition requires intuition and for the content and the justification of mathematical concepts and propositions.[15] Be those as they may, Brittan maintains that Kant's philosophy of mathematics faced some refutations – allegedly, almost all those who followed Kant discovered his off beam interpretation on *synthetic a priori* nature of mathematics including geometry and arithmetic, and spurred a blunt remark that the role of intuition in mathematics is erroneous.[16] Of all criticisms, it is the last that we should not miss to retort. Here, Brittan defended Kant by exposing the following exceptional defence in length:

> On one, our attention is drawn to the premises of mathematical inferences – the axioms, basic propositions, or principles of arithmetic and geometry. . . . Frege maintains that arithmetic is "analytic" in the sense that one can derive all of its truth in a logically rigorous fashion from the definitions of "zero". . . . Frege was

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[14] Gordon Brittan, "Kant's Philosophy of Mathematics," in *A Companion to Kant*, ed. Graham Bird (Oxford: Blackwell, 2006), 222; [Henceforth, *Companion*].

[15] Daniel Sutherland, "Kant's Philosophy of Mathematics and the Greek Mathematical Tradition," *The Philosophical Review* 113, no. 2 (2004): 158. The same observation was also put forward by Carol A. Van Kirk, "Synthesis, Sensibility, and Kant's Philosophy of Mathematics," *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association Volume One: Contributed Papers* (1986): 135-144. Philip Kitcher presupposed another subtlety in Kant's Philosophy of Mathematics. He wrote, "The heart of Kant's views on the nature of mathematics is his thesis that the judgments of pure mathematics are synthetic a priori" (23). He expounded this view in two theses: (1) The truths of pure mathematics are necessary, although they do not owe their truth to the nature of our concepts; and (2) The truths of pure mathematics can be known independently of particular bits of experience, although one cannot come to know them through conceptual analysis alone (ibid.). [Philip Kitcher, "Kant and the Foundations of Mathematics," *The Philosophical Review* 84, no. 1 (1975): 23-50].

[16] Brittan, *Companion*, 222.

> wrong in taking these axioms analytic; they can be denied without contradiction. . . . An appeal to "intuition" must also be made.[17]

Moreover, his second defence:

> . . . [We] are to concentrate not on the premises of mathematical inferences, but on the proof of procedures used to demonstrate their conclusions. This second line of interpretation has its source in Russell. Those who follow him maintain that the proof procedures furnished by monadic quantification theory (Aristotle's theory of the syllogism) . . . are not capable of establishing all of the conclusions demanded by mathematics [for an instance] those having to do with infinite series and the notion of continuity. It was for this reason that Kant was able to make reference to "extralogical" or "intuitive" consideration in mathematics.[18]

The first appeal can stand in itself by presupposing that not all mathematical laws or laws of logic cannot be denied analytically without contradiction. Nevertheless, these laws originated from intuition. To support the point, Kant argued in the *Critique* that *a synthetic proposition can of course be comprehended in accordance to the principle of contradiction*.[19] The second appeal, akin to the first, shows the role of intuition to mathematical proofs. Aristotelian syllogism for example was remodeled since Kant's time that gave way to the rise of new logical theories, i.e., modal logic and predicate logic. This remodeling was due to the syllogism's incapacity to answer the evolving demands of mathematics specifically those having to do with infinite series and continuity. With the pitfall of Aristotelian syllogism, Kant invested on what is 'extralogical' or by far 'intuitive' to address those issues. Without the intuitive capacity of the mind, mathematics and logic might not have progressed. This is where intuition becomes a major matter. Another example is causation. Normative causal law, as pioneered by Aristotle, Hume, Hobbes and others, states that there is a necessary connection between events in the determination that an event causes another event. G. E. M. Anscombe, on the one hand, refuted the claim intuitively, that *it must not be so*.[20] She said what if you have had a contact to a patient with a contagious disease and you ask the doctor whether you will get the same disease, what will be the doctor's response to you? Anscombe reminds us that: yes, there are laws of nature but *it is not [always] the case that it must be so*. The doctor will answer still: "may or may not be."

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[17] Ibid., 223.

[18] Ibid.

[19] Kant, *Critique*, B14 cited in Brittan, *Companion*, 223

[20] See G. E. M Anscombe, "Causality and Determination," in *The Collected Papers of G. E. M. Anscombe Volume 2: Metaphysics and the Philosophy of Mind* (Oxford: Basil Blackwell, 1981).

Kant, in this light, recognized the important role of intuition as merely extralogical, that is to say, healthy to the progression of the logico-mathematic laws of nature. It is noteworthy that for Kant, in the seventh section of *Prolegomena* (§7), he emphasized this role of intuition:

> But we find that all mathematical cognition has this peculiarity: it must first exhibit its concept in intuition, and do so *a priori,* in an intuition that is not empirical but pure. Without this mathematics cannot take a single step; hence its judgments are always *intuitive;* . . .This observation on the nature of mathematics gives us a clue to the first and highest condition of its possibility, which is that some pure intuition must form its basis, in which all its concepts can be exhibited or constructed, *in concreto,* and yet *a priori.*[21]

This peculiarity of mathematical cognition is no less than its intuitive nature brewing in inferences of mathematical truths and logical procedures. Intuition, in order to be understood as concept, needed to be a subject of synthetic appreciation and, at the same time, its *a priori* nature relates a mathematical object to the observer, in this case, a mathematician, that is affected by the object's presence. Apropos to this, mathematics presents all its concept to intuition by which Kant in §10 proposition of the *Prolegomena* asserts that Geometry is based upon the pure intuition of space and Arithmetic is merely representation of time (remember that 're-presentation', as Deleuze pointed, is a figurative synthesis of imagination, hence knowledge in which intuition, in turn, is essential).

To exemplify thus, let us turn into an excerpt from Kant's letter to K. L. Reinhold in 1789: "the mathematician can make no claim about an object without first pointing it out in intuition."[22] Obviously, like what the *Critique* and *Prolegomena* assert, mathematical claims are first "constructed" (as borrowed from Brittan) into intuition. Brittan, in order to show the important role intuition plays, cites a second letter, but this time, a letter from A. W. Rehberg to Kant: "Given that the understanding can create numbers at will, why is it not capable of thinking √2 (square root of 2) in numbers!"[23] Kant responded (but I will show here Brittan's simplification of Kant's complex response) showing that the square root of two is constructed in intuition specifically in geometrical representation of a

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[21] Immanuel Kant, *Prolegomena to Any Future Metaphysics*, trans. James W. Ellington (Indiana: Hackett, 1977), 25. We can also find in the *Critique* the same argument: "[All] mathematical concepts are not by themselves cognitions, except insofar as one presupposes that there are things that can be presented to us only in accordance with the form of that pure sensible intuition" (Kant, *Critique*, 254).

[22] Bratten, *Companion*, 232.

[23] Ibid.

diagonal of a unit square.[24] This implies that its numerical determination, at the outset, is not presented in thought (quite to say unthinkable) but as a representation of geometric space. Moreover, Kant said that one can determine the square root of two's numerical value by mathematical approximations say a calculation until you get its precise value. (akin to a trial and error computation).

In his philosophy of mathematics, Kant contends that mathematical judgments are derivable as *synthetic a priori judgments* by virtue of their representation in thought. Here, formal mathematical truths are presented to intuition and thus constructed for re-presentation in space as per geometry and in time, arithmetic. Hence, all objects of mathematics, be it arithmetic or geometric, as far as synthetic judgment is concerned, are intuited *a priori* and thus construct inference to represent a synthetic knowledge.

Introducing Ethnomathematics

In 1977, the Brazilian educator and mathematician Ubiratan D'Ambrosio first introduced the term *ethnomathematics* to cater a new lens to observe mathematical knowledge but this time not within the bounds of academe but the outside field of human life. In the course of forty years, ethnomathematics still scuffles to find its formal meaning. D' Ambrosio simply defines ethnomathematics as "mathematics practiced by cultural groups, such as urban and rural communities, groups of workers, professional classes, children in a given age group, indigenous societies, and so many other groups that are identified by the objectives and traditions common to these groups."[25] With this extent, ethnomathematics indubitably traces people residing in the margins, victimized by social and cultural exclusions and discriminatory barriers. Here, D' Ambrosio admitted that this field has undeniable political focus.[26] In the same line of argument, Monica Mesquieta defines ethnomathematics as

> [the] active process . . . of creating a possibility, doing, and making viable the allocation of specificity to new mathematical objects (new models, representation

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[24] Ibid.. The square root of two is a popular mathematical example. Its numerical value is 1.41421296 but one cannot think of it beforehand intuitively unless you memorize it. Legend has it that Pythagoreans secretly called it irrational but Hippasus of Metapontum learned and divulge it and thus killed. By then, it was known to be Pythagoras' constant. (See John Conway and Richard Guy, *The Book of Numbers* [New York: Copernicus, 1996], 25; See also Kurt von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum," *Annals in Mathematics* 46, no. 2 [1945]: 242-264).

[25] Ubiratan D' Ambrosio, *Ethnomathematics: Link between Traditions and Modernity* (Rotterdam and Taipei: Sense Publishers, 2001), 1; [Henceforth, *Ethnomathematics*].

[26] Ibid.

> and artefacts) as well as creating conditions where every human being can exercise his/her mathematical work through transformative productive forces.[27]

She further argues that "ethnomathematical practices propose an alternative model of human relations. . . . It proposes community models that oppose totalitarianism and challenge the myth of individualism."[28] Following Mesquita's definitions, we can find enlightening facts on what ethnomathematics has to offer. For her, ethnomathematics is an active process of human beings to create conditions where the possibility and viability of new mathematical objects can fit one's free mathematical work. In this sense, she recognized any human being's freedom in doing mathematics external to its academic nature that to a certain extent does not require any advanced academic knowledge. Ethnomathematics, as she believes, is no less than a community model of human relation doing a collective and shared understanding free from the exclusive and elite practice of academic hegemony. True to its political notion, Mesquieta and D'Ambrosio follow a nonconformist mathematical knowledge yet authentic in its scheme as it is discovered and practiced by people within their own communal space. It is a form of mathematics passed down to generation and abruptly learned without methodical nor pedagogical restrictions. Mathematics, as D'Ambrosio contends, should be a "life phenomenon."[29] In light of this princple, D'Ambrosio offered a general yet enlightening reflection that we must treat with philosophical importance,

> The adventure of the human species is identified with the acquisition of styles of behaviors and of knowledge to survive and transcend in the distinct environments it occupies, that is, in the acquisition of the natural, social, cultural, and imaginary environment (*ethno*) of explaining, learning, knowing, and coping with (*mathema*) modes, styles, arts and techniques (*tics*).[30]

D'Ambrosio recognizes the human species intuitive capacity to acquire modes of learning together with the instinct to survive in tone of its changing space and evolving time. The human species, furthermore, actively delineates its life phenomenon and progression that in a way leads it to discover new techniques of living that we can account as the core of any scientific discovery. Surprisingly we can find in Kant a similar position:

> Consciousness of itself (apperception) is the simple representation of the I and if all of the manifold in the subject were given self-actively through that alone, then the inner intuition would be intellectual. In human beings this consciousness

[27] Mesquieta, *Asphalt Children*, 64.
[28] Ibid.
[29] Ibid., 61.
[30] See D' Ambrosio, *Ethnomathematics*.

> requires inner perception of the manifold that is antecedently given in the subject and the manner in which is given in the mind without spontaneity must be called sensibility on the account of this difference. If the faculty for becoming conscious of oneself is to seek out (apprehend) that which lies in the mind, it must affect the latter, and it can only produce an intuition of itself in such a way, whose form, however, which antecedently grounds it in the mind, determines the way in which the manifold is together in the mind in the representation of time; there it then intuits itself not as it would immediately self-actively represent itself, but in accordance with the way in which it is affected from within, consequently as it appears to itself, not as it is.[31]

As consciousness ascertains within its reach any sensible object in the manifold of its experience, intuition provides apprehension and re-presentation of synthetic knowledge offered by sensibility. In other words, to borrow from D' Ambrosio's philosophical definition of ethnomathematics, the "acquisition of the natural, social, cultural, and imaginary environment of explaining, learning, knowing, and coping with modes, styles, arts and techniques." Ethnomathematics, in this sense, qualifies to the rigid requirement of the *Critique* to what constitutes mathematics derivable from the tenets of sensibility and intuition. Ethnomathematics, to assess in Kant's criterion, is a synthetic body of knowledge, amicable to intuition and thus, since it plots a discovery that is yet to happen in the course of history (by virtue of the word 'adventure'), is *a priori*.

Presently, we have answered the first two questions of this essay. Let us now venture on the third.

Kant and Precolonial Mathematics

The attempts of D'Ambrosio to singularize ethnomathematics as a cultural and life phenomena verified a changing *leitmotif* in the epistemological foundations of mathematics *per se*. Simply put, ethnomathematics has to inquire on its own position within epistemology thus, echoing Bill Barton's same inquiry whether ethnomathematics is "a precursor, parallel body of knowledge or precolonized body of knowledge."[32] Either/or, Kant already esteems the former; in other words, we can extend his *raison d'etre* in the *Prolegomena* about the possibility of mathematics to ethnomathematics by way of looking at the former as a 'precursor' and at the same time a 'parallel' body of knowledge. Before mathematics evolved into a serious academic pursuit, it originated primordially to

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[31] Kant, *Critique*, 189-90.

[32] Bill Barton, "Making Sense of Mathematics: Ethnomathematics is making sense," *Educational Studies in Mathematics* 31, 1 of 2 (1996): 201-33.

people whose minds were affected by objects in cognition in a certain way.[33] We can say that intuition is not solely exclusive for anyone who has academic merits on mathematics as a field but even to those people whose mathematical knowledge are naturally learned from the cultural and life phenomena.

Still, we are left with one question that goes in the same pole of Barton's inquiry of a 'precolonized body of knowledge.' Aye, there are ethnomathematical schemes today independent of the complexity of academic math. However, the word 'precolonized' is intriguing and by far controversial. Needless to say, the attribution of this field to epistemology, that is to say, to the production of knowledge, brings us to the important issue of who controls such theoretical production. To follow this contention, we will try to reflect on D' Ambrosio's claim that ethnomathematics poses a political question. Here, I will try to plot the narrative in the context of mathematics in early Philippine Society. We will also survey the vast array of Kant's *Critique* and *Prolegomena* to evaluate ethnomathematical practices in precolonial Philippines.

Ricardo Manapat's posthumous essay *Mathematical Ideas in Early Philippine Society* investigates the sophistication of mathematical practices in precolonial culture. However, this sophistication was also jacketed by multiple accusations. For example the Spanish Fray Gaspar de San Agustin in his 1703 *Compendio del arte de la lengua tagala* commented that "the Tagalogs are little suited for mathematics."[34] Later, as Manapat exposed, Fray de San Agustin released a harsher remark: "Tagalogs in counting are unreliable and bad mathematicians."[35] Fray Eladio Zamora, following his predecessors, evaluates 18th century Philippines in terms of "the small capacity of the [*indio*] for the sciences."[36] With these testimonies chronicled by early historians, indeed, precolonial mathematics in the Philippines reveals itself inscribed within an orientalist historiography. The depiction of old *Tagalogs* and *indios* as unequipped for scientific and mathematical works eclipses our authentic treatment of nature in our own way. But of course, these historians' evaluations do not matter at all if and only if we can prove, prior to colonization, that early Philippine society already used original and authentic mathematical schemes to formulate their day-to-day realities. In this nub, Manapat

[33] Kant is useful here: "In whatever way and through whatever means a cognition may relate to objects, that through which it relates immediately to them, and at which all thought as a means is directed as an end, is intuition. This, however takes place only insofar as the object is given to us, but this in turn, at least for us humans, is possible only if it affects the mind in a certain way" (Kant, *Critique*, 172).

[34] Ricardo Manapat, "Mathematical Ideas in Early Philippine Society," *Philippine Studies: Historical and Ethnographic Viewpoints* 59, no. 3 (2011): 293; [Henceforth, *Mathematical Ideas*].

[35] Ibid.

[36] Ibid., 293-94

elaborated a number of ethnomathematical practices that for the present writing, we will try to examine through the sites of Kantian intuition in the *Critique* and the *Prolegomena*.

Let us first devote a glance to an archaeological puzzle. Here, I will try to expose the existence of the 'mathematician' in question. The *Angono Petroglyphs*, a set of prehistoric carvings in the hills of Angono, south of Manila and contoured by a long mountainous range called Sierra Madre that ends in the largest lake in Asia Laguna De Bay, was dated as late Neolithic Period (3,000 years B.C.) – a thousand years earlier than the Greeks who first introduced complex mathematics. Manapat speculated that the petroglyphs suggest traces of basic geometric ideas that ostensibly are the earliest known mathematical discovery in our recorded history. Here, he cites Jesus Peralta, a famous anthropologist of the National Museum:

> As a general rule the drawings are of human figures, consisting of line incisions of circular or domelike heads with or without necks set on a rectangular V-shaped body. The arms, sometimes with digits, and the legs are also lineally executed, and are usually flexed. An inventory of the drawings produced a total of 127 figures clearly discernible integral units. This count excludes other incisions that comprise slashes, naturally occurring holes, scratches, pits, pockmarks and other surface alterations on the rockwall.[37]

Not to mention that in Peralta's observation, there were incisions that can be recognized as triangles, rectangles and circles and to a more surprising geometry, there is a complex of four triangles forming parallelogram.[38] The Neolithic mathematicians of the Angono Petroglyphs exposed a lineal geometric complexity which are abstractions representing an established body of knowledge. These abstractions made from lines and shapes are conscious apprehension of what is sensible in their space (synthetically), in other words, abstracted in their experiences in nature for example, lines from trees or curves of the clouds, that forms basic geometry in their thought (*a priori*). Albeit enlightening, this is where intuition comes into play with abstraction. Kant, surprisingly, explains it this way,

> Since the propositions of geometry are cognized synthetically *a priori* and with apodictic certainty, I ask: Whence do you take such propositions, and on what does our understanding rely in attaining to such absolutely necessary and universally valid truths? . . .Take the proposition that with two straight lines no space at all can be enclosed, thus no figure is possible, and try to derive it from the concept of straight lines and the number two; or take the proposition that a

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[37] Jesus Peralta, "Petroglyphs and Petrographs," *Kasaysayan* Vol. 2 (1998): 135; cited in Manapat, *Mathematical Ideas*, 295.

[38] Ibid., 295.

> figure is possible with three straight lines, and in the same way try to derive it from these concepts. All of your effort is in vain, and you see yourself forced to take refuge in intuition, as indeed geometry always does.[39]

Propositions of geometry for Kant are apodictically *synthetic a priori* but its synthetic grounding relies on *a priori* intuition. So, *a priori* intuition sets the structure for synthetic objects in cognition by which Kant warns that if there is no faculty for intuiting *a priori*, then how could one say that subjective (synthetic) cognition of three lines pertains to the formation of triangle itself?[40] Here, we must take Kant's warning seriously. Intuition, as underscored by *Prolegomena* in proposition §10, is fruitful when we know objects as they *appear* in our senses.[41]

Ergo, using the scheme Kant provided, the Angono Petroglyphs represent a *synthetic a priori* cognition. The space itself, represented by lines and geometrical shapes, are intuitions infused into an *imaginative* creation (we are talking here of Kant's figurative synthesis in B150/B151 of the *Critique*). Manapat in this context, writes,

> The prehistoric figures, furthermore, demonstrate that the Neolithic artists intuitively knew how to work with the notions of symmetry and proportion since the rock and cave drawings show a respect for the basic mathematical and aesthetical ideas of symmetry and proportion, as well as the more complicated idea of mathematical scaling, as seen in the successful resizing of the stone etchings from the actual, bigger figures of men they represent.[42]

The representation stands true of their sensibility, needless to say, of the objects appearing to them. In this light, Manapat also argues, "The Neolithic artists of Angono used lines to draw the figures which represented themselves and other members of their community."[43] No wonder Angono artists even today are known for their aesthetic crafts. This is where *imagination* in both Kantian and artistic sense configure into a powerful cognitive process.[44]

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[39] Kant, *Critique*, 187-88.

[40] Ibid., 188.

[41] Kant, *Prolegomena*, 27

[42] Manapat, *Mathematical Ideas*, 295-96.

[43] Ibid. 295.

[44] For Kant imagination "has to bring the manifold of intuitions into the form of an image" (A120). In this line, Heidegger said (following Kant, of course) that the power of imagination has meaning as a faculty of forming . . . imagination creates forms and provides the image" (Martin Heidegger, *Kant and the Problem of Metaphysics*, trans. Richard Taft [Indianapolis: Indiana University Press, 1997], 91).

Other than the Angono Petroglyphs, Manapat also cited that precolonial Philippine society practices geometric thinking in shipbuilding.[45] The eminent Spanish chronicler Fr. Francisco Colin observed the genius shipwrights of Catanduanes. The shipwrights can build a larger ship called *biroco* where ten to twelve smaller vessels can fit inside of it.[46] To engineer such naval technology, early Filipinos mastered geometric principles of convexity, concavity and the proper proportion between ship breath and length to conform the irregular tides of water movement. We can also account their exceptional knowledge of other factors in navigation such as wind, the wood type, and ships' strength especially in a typhoon-prone archipelago. Given these sensibilities, ancient shipbuilders must have employed intricate mathematics which surprisingly parallels with modern naval mathematics. Ancient shipbuilders, in accordance to their sensibilities, intuited geometric spaces equipping them synthetic skill to build ships prior to western naval influence.

Aside from ethnomathematical knowledge of geometric spaces, ancient Filipinos also exhibited exceptional skills in cosmology and arithmetic. For example, Manapat alludes to the historian Juan Francisco de San Antonio's 1738 *Cronicas* to illustrate early Filipino knowledge on cosmology and time:

> It is not known whether these natives divided the time in hours, days, weeks, months, or years, or made any other division of time. As this was necessary to them for the reckoning of their commerce, trade and contracts (in which they all engaged), they used for reckoning their times of payment, and for other transactions and business of their government – for the hours, state of the sun in the sky, the crowing of the cock, and the laying time of the hens, and several other enigmas which are still employed in the Tagalog speech. To keep account of the changing of season, they knew when it was winter or summer by the trees and their leaves and fruit. They knew of the division into months or years by moons.[47]

Rather than determining time in terms of standard clocks (which was only introduced in seventeenth century), early Filipinos, according to Manapat, hinge on subjective time modes that are similar to the Greek concept of *Kairos* or subjective time

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[45] Manapat, *Mathematical Ideas*, 297.

[46] Francisco Colin, *Labor evangelica de los Obreros de la Compania de Jesus en las Islas Filipininas*, ed. Pablo Pastells, SJ (Barcelona: Heinrich, 1904), 25; cited in Manapat, *Mathematical Ideas*, 297.

[47] Juan Francisco de San Antonio, "Cronicas de la Provincia de San Gregorio Magno," *The Philippine Islands*, 1493-1898, eds. Emma Blair and James Robertson (Cleveland: Arthur H. Clark, 1906). Quoted by Manapat, *Mathematical Ideas*, 300.

moments rather than *Kronos* or the strictly measurable time.[48] This ethnomathematical knowledge of time is consistent with Kant's. For Kant, time is given *a priori*, that is to say, "time is nothing other than the form of inner sense, i.e., of the intuition of our self and our inner state."[49] Henceforth, time for the precolonial Filipinos is a subjective determination presenting itself independent of appearances.

Manapat also recorded another interesting precolonial fact. The counting system of the early Filipinos did not subscribe to mathematical infinity; instead the last number according to records was a thousand *yuta* or one hundred million (100,000,000).[50] Old Tagalogs were perplexed of what goes beyond this number dismissing it as entirely inconceivable and mathematically void.[51] In this light, Fray Francisco Blancas de San Jose recorded this mathematical notion,

> One thousand *yota* and thousands of *yota* is not known. Instead, they say *sang bahala*, which means "What do I know? I leave it up to you [Bahala ka]. What can I do? Of these things one can no longer conceive.[52]

There are striking generalizations we can posit here. First, Old Tagalogs designate epistemological boundaries to what they know. Second, *sang bahala* (leaving it up to you) is a linguistic formula of leaving the inquiry to other and admitting the failure to conceive the answer in the moment of its utterance. These two points can invoke an ethnomathematical knowledge converging with some of Kant's thoughts.

Kant also included in his philosophy the rubble of a realm where knowledge by far is impossible. In the *Transcendental Doctrine of the Power of Judgment*¸ Kant made the revelation of such concept called *noumena*:

> The concept of the noumenon is therefore merely a boundary concept, in order to limit the pretension of sensibility, and therefore only of negative use. But it is nevertheless not invented arbitrarily, but is rather connected with the limitation of sensibility, yet without being able to posit anything positive outside of the domain of the latter.[53]

The existence of the noumenon gives limit to reason even to non-sensible intuition or intellectual intuition like the limit number *a thousand yuta* of the Old Tagalogs.

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[48] Ibid.,300.

[49] Kant, *Critique*, 180.

[50] Manapat, *Mathematical Ideas*, 331.

[51] Ibid.

[52] Francisco Blancas de San Jose, *Arte y reglas de la lenqua Tagala* (Bataan: 1610), 266; cited by Manapat, *Mathematical Ideas*, 332

[53] Kant, *Critique*, 350.

The limit gives room, however, to the existence of things in themselves which for Kant beyond our apprehension and cannot be known by human reason alone. Here, metaphysics meets its ultimate deadlock. In the "Transcendental Dialectic," Kant made a pronouncement that within the *noumena* are three metaphysical postulates that are incomprehensible, thus putting the tenets of metaphysics in a contentious crisis: God, Freedom, and Immortality.[54] Take note of the inclusion of God to postulates inconceivable to us. Here, the limit number as *sang bahala* (bahala ka) as an inconceivable thing for the old Tagalog makes a startling connection to Kant's suspension of knowledge about God:

> It is said that prior to the arrival of the Spaniards in the Philippines, the Malay race was the predominant race in the country and its attendant beliefs served as the backdrop of the predominant religion. This religion promoted the belief in *Bathala*, a kind and omnipotent being who is the provider of all things for man. Believing that whatever happens is *Bathala*'s will, the Filipinos of long ago heavily relied on this divine entity, fueling a risk taking behavior in the assurance that "Bathala will always take care" (Jocano, 1981, 5).[55]

Furthermore, the word *Bathala*, in resonance with Filipino belief in divine providence, evolved syntactically into *Bahala na*. In this vein, *bahala na* suggests a fatalistic tendency to leave everything to a more powerful being beyond us. *Sang bahala*, as representation of the mathematical infinite, evolved linguistically and semantically into Tagalog's *Bathala* that brings us to a similar God postulate in Kant – a *noumenal* object, to borrow from the popular opening line of the *Critique*, 'transcending the capacity of human reason.'

Conclusion

Precolonial ethnomathematics in the Philippine setting is justified as far as Kantian philosophy is concerned. Mathematical knowledge through intuition and sensation undeniably establishes a proof that our Filipino ethnomathematics exists and as read in Kantian lens, it is profound and integral especially in a history outshone by unfair and biased historiography among colonial historians. Interestingly, enumerated ethnomathematical practices in the Philippines including the earlier Angono Petroglyphs, shipbuilding technology, subjective time conception and limit number not only bring us to political inquiries but also to our own epistemological quests. In the final analysis, this paper does not entail the usage of a western rubric to evaluate eastern knowledge. On

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[54] Ibid., 117;

[55] See Ma. Ligaya Manuel Manguito and Mendiola Teng-Calleja, "*Bahala na* as an Expression of the Filipino's Courage, Hope, Optimism, Self-efficacy and Search for the Sacred," *Philippine Journal of Psychology* 43, no. 1 (2010): 1-26.

the contrary, Kant's critical thoughts in his *Critique* and *Prolegomena* as a tool to read Philippine ethnomathematics proved, then and again, an insightful generality – the possibility of philosophy to articulate a shared life phenomenon.

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